

**ANALYSIS OF CAUSAL RELATIONS BETWEEN
INTERCOMMUNICATING NODES**

Inventors

Marcos K. Aguilera

Jeffrey C. Mogul

ANALYSIS OF CAUSAL RELATIONS BETWEEN INTERCOMMUNICATING NODES

FIELD OF THE INVENTION

[001] The present disclosure generally relates to analyzing the relationships between messages sent between nodes.

5

BACKGROUND

[002] Many commercially-important systems, especially Web-based applications, are composed of a number of communicating components. These systems are often structured as distributed systems, with components running on different
10 processors or in different processes. For example, a multi-tiered system may process requests from Web clients that flow through a Web-server front-end and then to a Web application server. The application server may then call a database server, for example, or other types of services such as for authentication, name service, credit-card authorization, or customer relationship management or other support functions.

15 [003] Distributed systems can be difficult to debug, especially when users experience poor performance. Diagnosing performance issues is even more difficult in distributed systems if the constituent components are composed of "black-box" components. For example, some distributed systems may be constructed from software from many different, and perhaps competing, vendors, and the source code of the different
20 components may be unavailable. Without more than a high-level understanding of the functions provided by the various components, and without the information that could be learned from examination of the source code, selecting a component to begin investigating may involve guesswork and result in wasted time.

[004] The business model under which distributed systems are sold and
25 deployed also contributes to the difficulties associated with addressing performance problems. Enterprises often buy complex systems as complete, customized packages from solutions vendors. Solutions vendors may be pressured to deliver complex component-based systems without the expense of highly-skilled, experienced programmers. While modestly-skilled programmers can design and construct such systems, they may lack the
30 expertise to debug performance problems efficiently. Vendors of individual components may provide training and support for solving performance problems within the

components, but not necessarily support for solving performance problems when components from other vendors are involved. Thus, whole-system performance debugging may require either an inordinate amount of time or the services of expensive and hard-to-find systems integration experts. The present invention may address one or
5 more of the above issues.

SUMMARY

[005] The various embodiments of the invention support determining causal relations between a plurality of intercommunicating nodes. Communications between the nodes may be described by input trace data. The trace data may include for each message
10 sent between nodes a timestamp that indicates a time at which the message was sent, a source identifier that identifies a node from which the message was sent, and a destination identifier that identifies a node to which the message was sent. For each of one or more nodes, a determination is made as to whether one or more causal relations exist between a
15 first set of messages destined to the node and a second set of messages sourced from the node and destined to at least one other node. A causal relation may exist as a function of a probability distribution of delay values that are differences between timestamps of messages in the second set and timestamps of messages in the first set. From the nodes and causal relations a processor-readable representation is generated.

20 [006] It will be appreciated that various other embodiments are set forth in the Detailed Description and Claims which follow.

BRIEF DESCRIPTION OF THE DRAWINGS

[007] FIG. 1 is a block diagram of an example multi-tier data processing arrangement in which an example causal path is illustrated in accordance with
25 embodiments of the invention;

[008] FIG. 2 is a graph of the example causal path from FIG. 1;

[009] FIG. 3A illustrates an example set of trace messages including two example subsets;

30 [0010] FIG. 3B illustrates part of a graph with an edge representing the causal relation inferred from FIG. 3A in accordance with embodiments of the invention;

[0011] FIG. 4 illustrates an example process for generating a graph in accordance with various embodiments;

[0012] FIG. 5 illustrates an example process for processing a node in accordance with embodiments of the invention;

[0013] FIG. 6 illustrates an example process for inferring causation between sets of messages destined to a node and sets of messages sourced from the node in accordance with embodiments of the invention;

[0014] FIG. 7 illustrates an example process for finding a correlation between messages destined to a node and sets of messages sourced from the node in accordance with embodiments of the invention;

[0015] FIGs. 8A and 8B illustrate indicator functions that signal occurrences of messages versus time for messages destined to a node and sets of messages sourced from the node respectively;

[0016] FIG. 8C contains a graph of a cross-correlation function of the indicator functions of FIGs. 8A and 8B.

[0017] FIG. 9 illustrates an alternative example process for inferring causation between sets of messages destined to a node and sets of messages sourced from the node in accordance with embodiments of the invention;

[0018] FIG. 10 illustrates an example process invoked by the process of FIG. 9 for selecting a relevant set of destination nodes based on a cross-correlation between two sets of messages in accordance with embodiments of the invention;

[0019] FIG. 11 illustrates another alternative example process for inferring causation between sets of messages destined to a node and sets of messages sourced from the node in accordance with embodiments of the invention;

[0020] FIG. 12 illustrates another alternative example process for inferring causation between sets of messages destined to a node and sets of messages sourced from the node in accordance with embodiments of the invention; and

[0021] FIG. 13 illustrates other embodiments for selecting sets of relevant destination nodes in accordance with embodiments of the invention.

DETAILED DESCRIPTION

[0022] The various embodiments described herein infer causal relations of messages sent between nodes, where the causal relations may assist in locating performance problems. The nodes may represent host computer systems in a distributed data processing arrangement, computer processes, threads, active objects, disk drives or

various combinations thereof. The inner functions of the constituent nodes need not be apparent to infer the causal relations. The nodes and communication relationships may be represented as a graph, with vertices in the graph representing the nodes and edges between the vertices representing communications between the nodes.

5 [0023] The causal relations may be inferred from traced communication information. The nodes may be determined from the traced communication information and may be represented as vertices in a graph. An edge may be added to the graph to connect two vertices that represent nodes that communicate, as indicated by the traced communication information. A causal relation may be inferred between a source node and
10 a destination node from delays between messages destined to the source nodes and messages from the source node to the destination node. Each inferred causal relation may be represented as a delay value that is associated with the edge from the source node to the destination node. The delay values associated with the edges may indicate potential performance problems that merit further investigation.

15 [0024] It will be recognized that the various embodiments do not require trace information resulting from the target distributed system implementing remote procedure calls (RPCs). Furthermore, the embodiments work with the trace information that is available, even though the trace data may be less than exhaustive due to starting and stopping of nodes, missed messages, and tracing limitations during peak activity. The
20 processes also are adapted to handle trace information from nodes in which the reference clocks are not synchronized.

 [0025] FIG. 1 is a block diagram of an example multi-tier data processing arrangement in which an example causal path is illustrated. The example tiers include clients 102a-e, which communicate with tier-3 nodes 106a-c. The tier-3 nodes
25 communicate with tier-2 nodes 110a-c, and the tier-2 nodes communicate with one another and communicate with tier-1 nodes 114a-d. The particular functions provided by the nodes in the various tiers are application dependent. For example, one or more data centers may host web servers that are tier-3 nodes 106a-c, application servers that are tier-2 nodes 110a and 110b, an authentication server that is tier-2 nodes 110c, and database
30 servers that are tier-1 nodes 114a-d. Even though it is not shown, it will be appreciated that the network components, such as switches and routers could also be nodes that are considered in a causal path.

[0026] The solid, directional lines between the nodes represent communication activity between the nodes. For example, line 120 represents bi-directional communication activity between client 102e and tier-3 node 106c. The communication events between nodes may be referred to as *messages*. From a collection of messages that are exchanged between the nodes, a causal path may be inferred. For example, dashed line 132 illustrates a hypothetical causal path. Inferring a causal path assumes that there is some causality between messages sent between different nodes. For example, a message directed to a first node may result in one or more messages being sent from the first node to one or more other nodes.

[0027] The example causal path 132 represents a possible scenario in which client 102e sends a message to tier-3 node 106c. In response, tier-3 node 106c sends a message to tier-2 node 110c, which in turn sends a message to tier-1 node 114d. Node 114d responds with a message back to node 110c, which returns a message to node 106c. Node 106c sends a message to tier-2 node 110b, which in turn sends a message to tier-1 node 114a. Messages are then sent back up the tiers from node 114a to node 110b, from node 110b to node 106c, and from node 106c to client 102e. It will be appreciated that multiple causal paths may be inferred from an input set of messages, even though only one causal path is illustrated in FIG. 1.

[0028] The input set of messages (*message trace*) from which causal paths may be inferred may be compiled from a number of sources, for example, passive network monitoring (for communication edges that flow between computers), kernel instrumentation, middleware instrumentation, or even application instrumentation.

[0029] The information of interest in the trace messages includes a timestamp, a source identifier, and a destination identifier. The timestamp indicates the time at which the message was sent, the source identifier indicates the node from which the message was sent, and the destination identifier indicates the node to which the message was sent. The timestamps of different source nodes (*sources* for brevity) need not have the same time reference. In some distributed systems, the collecting of trace messages may be distributed along nodes of the system so that different sources are monitored by different entities having local clocks. These clocks need not be synchronized with each other, the clocks need only have approximately the same rate, which is the case if they accurately measure intervals of real time.

[0030] FIG. 2 is a graph of the example causal path 132 from FIG. 1. Each of the vertices is labeled with the reference number of the represented node from FIG. 1. For example, the first vertex in the graph is 102e, which represents client node 102e from FIG. 1. Edge 142 connects vertex 110c to vertex 114d and represents a causal relation inferred between messages arriving at node 110c and messages from node 110c to node 114d. A node may be represented by more than one vertex in a graph depending on the inferred casual relations. For example, each of nodes 106c, 110b, and 110c is represented with two vertices in the graph because two causal relations are inferred for each of the nodes.

[0031] Whether a causal relation is inferred between messages from a source node to a destination node and messages arriving at the source node depends on a probability distribution of differences between timestamps of messages from the source to the destination and timestamps of messages arriving at the source. For example, in one embodiment if some number of messages from the source to the destination have timestamps indicating approximately equal delays relative to the timestamps of some messages received by the source, a causal relation is inferred between the source and the destination. It will be appreciated that other embodiments may use different probability distributions depending application-specific patterns of communication between nodes. In one embodiment, the causal relation is represented with an edge in the graph, and the most frequent delay is associated with the edge. For example, the edge from vertex 114d to vertex 110c has an associated delay denoted by d_4 .

[0032] FIGs. 3A and 3B illustrate the inference of a causal relation between nodes from a set of traced messages 160. FIG. 3A illustrates an example set of trace messages, with two example subsets shown in blocks 162 and 164, respectively. Subset 162 includes trace messages destined to node 110b, and subset 164 includes trace messages from node 110b and destined to node 114a.

[0033] The purpose of presenting the example timestamps is to illustrate a causal relation that may be inferred from messages destined to a node and messages sourced from the node and destined to another node. It will be appreciated that many additional trace messages would likely be present in a complete set of trace data. However, only messages of interest are shown in order to simplify the figures.

[0034] From the timestamps it may be observed that each value in subset 164 is offset by approximately 3.000 seconds from one of the values in subset 162. Because there are a number of corresponding messages in subsets 162 and 164 for which this

relation is true, it may be inferred that for at least some messages there is approximately a 3.000 second delay between the occurrence of a message destined to node 110b and the occurrence of a message sourced from node 110b and destined to node 114a. In other words, an inference may be made that some messages to node 110b cause messages to be generated to node 114a with a delay of approximately 3.000 seconds.

[0035] FIG. 3B illustrates part of a graph having an edge from vertex 110b to vertex 114a. The edge from vertex 110b to vertex 114a represents a causal relation inferred from the subsets 162 and 164 of trace messages. The value 3.000 seconds is associated with the edge from vertex to represent the approximate delay found to occur between the timestamps of subsets 162 and 164.

[0036] FIGs. 4-13 describe various embodiments of processes for inferring causal relations between nodes and generating a graph that represents the causal relations. FIG. 4 illustrates an example process 200 for generating a graph in accordance with various embodiments. The process generally considers messages targeted at a given node and the messages sourced from that node to find the likely candidate nodes for the next node in the path and then recurses for each candidate node. The next candidate nodes are found by the aggregation of multiple messages, rather than by examining one event at a time. The process seeks trends in this aggregation, while ignoring messages that are inconsistent with the trend.

[0037] The output of the process is a graph representation (*output graph*) of the causal paths that are inferred. The output graph begins empty (step 202). T_i is the subset of trace messages with source node i as determined from the trace of all messages (step 204).

[0038] The process begins with an initial node, represented by variable `initial_node`, and a vertex is added to the output graph (step 206) to represent the initial node. The process then considers the messages sourced from the `initial_node` (represented by $T_{\text{initial_node}}$). For each destination node, j , found among these events (step 208), there is a causal relation between the `initial_node` and j , and an edge is added from the vertex $x_{\text{initial_node}}$ to a vertex that represents node j (step 212). This edge is labeled with a zero delay because, by definition, the initial node does not delay messages.

[0039] The process then determines how the path continues from node j by invoking the `Process_Node` process (step 214), inputting node j , vertex x_j , and V (from step 210), which is the set of messages in $T_{\text{initial_node}}$ with having a destination of node j .

The Process_Node process 250 (FIG. 5) calls the Find_Caused_Messages process 300 (FIG. 6) with inputs V and T_j to find the subset of messages of T_j that are caused by the messages of V (step 252). Rather than return all the caused messages in a single group, the Find_Caused_Messages process splits those messages into multiple subsets. More precisely, Find_Caused_Messages returns a list, O_1, O_2, \dots, O_m , where each O_i represents the caused messages events in T_j that have a common delay and destination node. Each element O_i in this list has three fields: $O_i.messages$ contains messages of T_j , $O_i.delay$ is the common delay for the messages in $O_i.messages$, and $O_i.node$ is the common destination node for the messages in $O_i.events$. Then, for each such O_i , Process_Node creates a new vertex for $O_i.node$, and adds an edge to that vertex from x_j labeled $O_i.delay$ (steps 256, 258). Finally, Process_Node calls itself recursively to find out how the path continues from $O_i.node$ (step 260).

[0040] FIG. 6 illustrates an example process 300 (Find_Caused_Messages) for inferring causation between sets of messages destined to a node and sets of messages sourced from the node. The input to the process includes sets of messages V and Z . All the messages in V have the same destination node, j , and all messages of Z have j as the source node. The Find_Caused_Events process may be understood by way of example.

[0041] If for some duration of time d (*causal delay*), there are some number of messages in Z that appear exactly after time d of some message in V , then it may be inferred that those messages in Z are caused by those messages in V . The process attempts to find these causal delays (steps 304, 306), and for each causal delay found (step 308) the process finds the set Z_0 of events in Z corresponding to events in V time shifted by d (step 310). The time shift may be forward ($d \geq 0$) or backward ($d \leq 0$). A backward shift may occur if, during the gathering of the trace, there was lack of synchronization among clocks of different nodes. The process separates the messages in Z_0 by destination node: for each such node, a new list element O_i is created (steps 312, 314, 316).

[0042] The Find_Caused_Events process invokes the Find_Correlation process 350 of FIG. 7 to find a cross-correlation between messages destined to a node and subsets of messages sourced from the node. The process begins by converting input message sets V and Z into respective indicator functions $s_1(t)$ and $s_2(t)$, respectively (steps 352, 354). In one embodiment, indicator function $s_1(t)$ is defined as:

$$s_1(t) = 1 \text{ if } V \text{ has a message at time } [t - \epsilon, t + \epsilon] \\ 0 \text{ otherwise}$$

where ε is a selected small, fixed constant. The events of Z are similarly converted into indicator function $s_2(t)$. FIGs. 8A and 8B illustrate example graphs of indicator functions.

[0043] If a causal delay d exists, then $s_2(t)$ will include a copy of $s_1(t)$ shifted in time by d . Any causal delay(s) between $s_1(t)$ and $s_2(t)$ is characterized by computing a cross-correlation function (step 356). In one embodiment, the cross-correlation function $C(t)$ is the convolution of s_2 and the time inverse of s_1 . In general terms, $C(t)$ will have a spike at d if and only if $s_2(t)$ contains a copy of $s_1(t)$ time shifted by d .

[0044] It will be appreciated that the time inverse of a function $s(t)$ is $s(-t)$. The convolution of two functions $f(t)$ and $g(t)$ is another function denoted $f \otimes g(t)$, which is defined as:

$$f \otimes g(t) = \int_{-\infty}^{+\infty} f(u)g(t-u)du.$$

[0045] The discrete version of the convolution is:

$$(f \otimes g)_i = \sum_{j=-\infty}^{+\infty} f_j g_{i-j}.$$

[0046] If f is replaced with $s_2(t)$ and g is replaced with $s_1(t)$, the following formula results and is used for $C(t)$ in one embodiment:

$$C(t) = \int_{-\infty}^{+\infty} s_1(u-t)s_2(u)du.$$

[0047] The discrete version is:

$$C_i = \sum_{j=-\infty}^{+\infty} (s_1)_{j-i} (s_2)_j.$$

[0048] Returning now to the Find_Caused_Messages process 300 of FIG. 6, once the cross-correlation function is determined (step 304), the process finds the positions of the spikes in the cross-correlation function (step 308). In an example embodiment, the spikes are determined by computing the mean and standard deviations of C , and defining a spike to exist if it is N_1 standard deviations above the mean, where N_1 is a small fixed constant, for example, 1. The spike may gradually increase before reaching the peak. To avoid multiple detections of the same spike in nearby points in one embodiment, a spike must fall below N_0 standard deviations before another spike is detected, where $N_0 < N_1$ and is another small constant value, for example, 0.8. Other methods of finding spikes in a function may also be suitable.

[0049] Each spike may yield many candidate points. These candidate points include points that are N_1 standard deviations above the mean to a point N_0 deviations below the mean. Among these candidate points, the point with the largest value is selected to represent the spike. Because the position of the spike is the value of interest, the value d such that $C(d)$ is a spike is sought.

[0050] FIGs. 8A and 8B illustrate indicator functions that signal occurrences of messages versus time for messages destined to a node and sets of messages sourced from the node respectively. FIG. 8C contains a graph of a cross-correlation function of the indicator functions of FIGs. 8A and 8B. FIG. 8A illustrates the indicator function $s_1(t)$, and FIG. 8B illustrates the indicator function $s_2(t)$. It may be observed that $s_2(t)$ includes a version of $s_1(t)$ that is time shifted by d .

[0051] The cross-correlation function of $s_2(t)$ and $s_1(t)$ is shown in FIG. 8C. The graph illustrates that at a delay of d , there is a spike. The spike shows that some number of messages in Z are time shifted by d relative to a corresponding number of messages in V .

[0052] Also, for a given time t' , $C(t')$ approximates the amount of overlap between s_1 and s_2 when s_1 is shifted by t' . Overlap refers to the times at which both s_1 shifted by t' and s_2 are non-zero. For example, in FIG. 8C, if $t'=0$ then there is no overlap because there are no times when both s_2 and s_1 are non-zero. If t' is a small value then hump 372 of s_2 (FIG. 8B) will overlap with hump 374 of s_1 (FIG. 8A) because when s_1 is shifted right by a slight amount, there will be times when both s_2 and s_1 are simultaneously non-zero. This overlap is maximum when $t' = d$, which is where all humps of s_1 overlap with all hump of s_2 .

[0053] FIG. 9 illustrates another embodiment of a Find_Caused_Messages process 400. Relative to the Find_Caused_Messages process 300 of FIG. 6, the Find_Caused_Messages process 400 of FIG. 9 first splits the set of messages Z into smaller subsets based on destination node, and then finds the cross-correlation between V and each of the smaller subsets.

[0054] The Find_Related_Nodes process 450 (FIG. 10) is invoked (step 404) to return a set of destination nodes that might be of interest. Because Z may have a very large number of destination nodes, the overall process may be slow. Therefore, rather than processing every destination node, a subset of destination nodes is selected based on the likelihood of being relevant.

[0055] For each destination node j of interest (step 406), the process selects a subset Z_0 of messages from Z with j as the destination node (step 408). The Find_Correlation process 350 is then invoked with V and Z_0 (step 410), and the positions of the spikes in the cross-correlation function are found (step 412). For each spike position d (step 414), a subset Z_1 of messages is constructed to include messages in Z_0 having timestamps equal to timestamps of messages in V shifted by d . A new list element O_i is created for the destination node, delay value, and subset of messages (step 420).

[0056] FIG. 10 illustrates an example Find_Related_Nodes process 450 for selecting a relevant set of destination nodes based on a cross-correlation between two sets of messages. The process is similar to the Find_Caused_Message process 300 of FIG. 6. The Find_Related_Nodes process finds a cross-correlation function (step 454) and finds spikes in the cross-correlation function (step 456). For each spike position d (step 458), the process gets the messages in Z having timestamps equal to timestamps in V shifted by d (step 460) and collects in the Nodes set the destination nodes of the messages (step 464).

[0057] FIG. 11 illustrates yet another embodiment of a Find_Caused_Messages process 500. The accuracy of the Find_Caused_Messages process may be further improved by searching for only one spike at a time rather than multiple spikes. The Find_Caused_Messages process 500 searches for only the position d of the largest spike (the maximum value of $C(t)$).

[0058] The Find_Caused_Messages process 500 begins (steps 502, 504, 506, 508) in a manner similar to the Find_Caused_Messages process 400 (FIG. 9). The process then tracks a set V_0 of messages, which is initially set to V (step 510). When the number of messages in the smaller of V_0 and Z_0 reaches less than a selected minimum size (for example, 20 or 30 messages) *While* loop (step 512) is exited (step 514) and the list O_1, O_2, \dots, O_i is returned. The process finds a cross-correlation function for V_0 and Z_0 (step 516). If the maximum value of the cross-correlation function is not prominent, the loop is exited (step 518). A prominent value may be defined to be that the value is a selected number of standard deviations, for example 2 or 3, above the mean.

[0059] The process then establishes d as the position of the maximum of the cross-correlation function (step 520). Z_1 is assigned the subset of messages in Z_0 having timestamps equal to timestamps of messages V_0 in shifted by d (step 522). V_1 is assigned the subset of messages in V_0 having timestamps equal to timestamps of messages Z_1 in

shifted by $-d$ (step 524). The list element O_i is then updated (step 526), and messages in V_1 and Z_1 are removed from sets V_0 and Z_0 , respectively (step 528).

[0060] FIG. 12 illustrates yet another embodiment of a Find_Caused_Messages process 550. The previously described Find_Caused_Messages processes 300, 400, and
 5 500 may be improved further to address the following issue. Suppose that when a message arrives at a node j , node j delays by either d_1 or d_2 , with probability of $\frac{1}{2}$ each, before generating a message to a destination node. Then only about half as many messages that arrive in node j leave node j for each delay d_1 or d_2 . Thus, when the Find_Caused_Messages process reaches node j , it will tend to return subsets O_i that have
 10 half as many events as in V . If there are many nodes similar to j , then as the process progresses further in the causal path, there will be fewer and fewer messages available for consideration. With fewer messages to consider, accuracy may be impaired.

[0061] A second issue is that the output graph for node j will have two outgoing edges, one for d_1 and another for d_2 . If there are many nodes like j , an exponential
 15 increase in the number of nodes may result, which will likely clutter the output graph.

[0062] The Find_Caused_Messages process 550 addresses these issues by merging together the sets Z_1 for different delays (steps 562 and 580) and then creating a single O_i for all delays. It will be appreciated that the definition of O_i is slightly modified to encompass a set of delays rather than a single delay (steps 564 and 586). The
 20 consequence is that each edge of the graph will be labeled with a set of one or more delay values rather than a single delay value. It can be seen that the remaining steps of the Find_Caused_Messages process 550 are the same as corresponding steps in the Find_Caused_Messages process 500 (FIG. 11).

[0063] FIG. 13 illustrates another embodiment for selecting sets of relevant
 25 destination nodes according to a Find_Related_Nodes process 600. It will be appreciated the teachings of the embodiment of the Find_Caused_Messages process 500 (FIG. 11) may be adapted to improve the Find_Related_Nodes process as illustrated by process 600. A compromise between speed and accuracy may be obtained by finding multiple destination nodes that occur frequently rather than finding just one frequently occurring
 30 destination node.

[0064] The set of *Nodes* to be returned begins as an empty set (step 602), and V_0 and Z_0 are initialized to the input sets V and Z , respectively. A processing loop (step 606) is performed until certain exit conditions occur. The processing of steps 608, 610, 612,

614, and 616 is similar to the processing described for steps 514, 516, 518, 520, and 522 in the Find_Caused_Messages process 500 (FIG. 11).

[0065] Node i is established as the node that is the most frequent destination in the Z_1 subset of messages (step 618). Node i is then added to the output set of *Nodes* (step 620), and the Z_2 subset of messages is assigned the messages in the Z_1 subset of messages having as the destination node i (step 622). A V_2 subset of messages is established as the messages in V_0 having timestamps equal to timestamps of messages in Z_2 shifted by $-d$ (step 626). Messages in V_2 and Z_2 are then removed from sets V_0 and Z_0 , respectively (step 626). Upon exit from the loop, the set of *Nodes* is returned (step 628).

[0066] The following discussion describes various additional alternative embodiments. In one alternative embodiment, discrete indicator functions are provided to function with discrete variables rather than continuous variables in order to improve processing efficiency. A time quantum, μ , is chosen and t is treated as an integer that represents multiples of μ . The various parts of the processes that use t may be modified accordingly. The indicator functions $s_1(t)$ and $s_2(t)$ are redefined as follows:

$$s_1(t) = \begin{cases} 1 & \text{if } V \text{ has a message during times } [t\mu, (t+1)\mu] \\ 0 & \text{otherwise} \end{cases}$$

where t is an integer.

[0067] The s_1 indicator function may be alternatively defined as:

$$s_1(t) = \text{number of messages in } V \text{ during times } [t\mu, (t+1)\mu]$$

where t is an integer. Yet another alternative is to define the s_1 indicator function as:

$$s_1(t) = \text{square root of the number of messages in } V \text{ during times } [t\mu, (t+1)\mu]$$

where t is an integer.

[0068] It will be appreciated that the s_2 indicator function may be similarly defined using Z instead of V as the message set. For either alternative, the indicator functions may be represented by arrays in which each cell in the array represents a time quantum and the value of the cell represents the number of messages in the time quantum.

[0069] Another embodiment addresses the situation where the delays are not uniform. The method involves convolving the indicator functions with an arbitrary curve that may model the variance of delays. For example, the curve could be a Gaussian function or a triangle centered at the origin. This change may be implemented after step 354 in the Find_Correlation process 350 (FIG. 7), by adding the following code:

$$s_1 := \text{convolution}(s_1, \text{curve})$$

$$s_2 := \text{convolution}(s_2, \text{curve})$$

[0070] If the curve is a triangle or a Gaussian, the width of the triangle or the variance of the Gaussian can be input parameters to the process, and the chosen value may depend on the maximum variance expected for the delays. Application of this process may smooth spikes in the indicator functions with the expectation that there will be spikes in the cross-correlation of $s_1(t)$ and $s_2(t)$ even when there are variations in the delay.

[0071] Variances in the delay may also be addressed by considering messages having delays that are close to one another when collecting subsets of messages in the Find_Caused_Messages processes 300, 400, and 400. For example, various steps in these processes are similar to:

$$X := \text{messages in } Y \text{ having timestamps equal to timestamps of messages in } Z \text{ shifted by } d$$

where X , Y , and Z vary according to the particular version of and location within the process. The respective steps may be modified as:

$$X := \text{messages in } Y \text{ having timestamps within } \alpha \text{ of timestamps of messages in } Z \text{ shifted by } d$$

where α is a parameter with a value selected according to the magnitude of the delay variations.

[0072] In various other embodiments, the processes are adapted to address undesirable causal paths that may be found. That is, some causal paths may be inferred from the trace messages when in fact there is no causality. This may be problematic if the number of undesirable paths is so large that the useful paths are obscured. One way to deal with undesirable paths is to remove paths that are caused by a small number of messages. An alternative approach is to weight the edges based on the number of messages from which a causal relation is inferred.

[0073] To avoid undesirable paths, the Process_Node process 250 (FIG. 5) may be modified to avoid processing subsets O_i that have too few events. This may be accomplished by conditioning on the number of messages the creation of a vertex and the recursive invocation of the Process_Node process 250 (FIG. 5). Specifically, steps 258 and 260 of the Process_Node process 250 may be changed as follows:

if $|W| > \text{MinSize}$ then

add a new vertex x_k labeled k and edge (x_j, x_k) labeled d in the output graph

Process_Node (k, x_k, W)

where MinSize is a constant, for example, in the range of 20 to 30.

[0074] In another embodiment, the Process_Node process 250 may be adapted to prevent formation of paths that may be undesirably short. For example, if the traced
 5 messages indicate a causal path $A \rightarrow B \rightarrow C \rightarrow B \rightarrow A$, then the causal path $A \rightarrow B \rightarrow A$ is also likely to be detected because there are messages from B to A after some reasonably regular delay of the messages from A to B . This situation may be addressed by discarding edges that immediately return to the previous node in the path. However, it is desired for the process to still allow, in the example above, the edge $C \rightarrow B$. Thus, such an edge is
 10 allowed to return to the previous node only if there are no other edges leaving the current node. The Process_Node process 250 may be adapted to take an extra input, prev_node:

Process_Node(prev_node, j, x_j, V)

Step 260 is modified to pass this extra parameter to Process_Node:

Process_Node(j, k, x_k, W)

15 Step 214 is similarly modified:

Process_Node(dummy, j, x_j, V)

where *dummy* is the name of a node that does not exist, since there is no previous node.

Lastly, steps 258 and 260 of the Process_Node process are modified as follows:

if ($k \neq \text{prev_node}$) or
 20 ($O_i.\text{node is equal to } k \text{ for every } i$) then
 create vertex x_k labeled k and edge (x_j, x_k) labeled d in the output
 graph
 Process_Node (k, x_k, W)

[0075] Negative time shifts may also be found if the cross-correlation function C
 25 has a spike at a negative value of t . This may be a result of statistical chance or different time references in the timestamps and may result in inaccuracies in the output graph. If it is known that the time references were synchronized during collection of the trace messages, then negative time shifts may be ignored because the negative values will be due to statistical chance. The Process_Node process 250 may be modified to condition
 30 execution of steps 258 and 260 on $d \geq 0$. That is, if $d \geq 0$ then steps 258 and 260 are executed.

[0076] The output graph may have cycles that repeat many times if the trace messages exhibit periodic behaviors. For example, suppose that there is a message from A

to B every 1 second for 1 hour, and similarly from B to C and from C to A . Then the process may infer that the messages from A to B are causing the messages from B to C , which in turn are causing the messages from C to A , which cause events from B to A , and so on. The output graph would have a long cycle: $A \rightarrow B \rightarrow C \rightarrow A \rightarrow B \rightarrow C \dots$ This
5 output is undesirable because it might obscure useful information or cause the process to execute for a very long time since the run-time is a function of the size of the output graph.

[0077] An example embodiment to address this situation is to impose a maximum number of times, M , that a node can repeat itself in a path, where M is a small constant, for example, 4 or 5. This may be implemented by adapting the Process_Node
10 process 250 to condition execution of steps 258 and 260 on whether node k has appeared fewer than M times in the path from $x_{\text{initial_node}}$ to x_j .

[0078] In another embodiment, weights may be associated with edges so that relevant information may be later separated from irrelevant information. This may be implemented by changing step 258 of the Process_Node process 250 to add a weight to the
15 edge. The weight may be either a function of the number of messages in W or a function of the quality of the spike that originated the edge. "Quality" may be defined in a variety of ways, for example, by the number of standard deviations separating the mean and the spike. The weight may alternatively be a function of both the number of messages in W and the quality of the spike.

[0079] Those skilled in the art will appreciate that various alternative data
20 processing and or processor arrangements would be suitable for hosting the processes and data structures of the different embodiments of the present invention. In addition, the processes may be provided via a variety of computer-readable media or delivery channels such as magnetic or optical disks or tapes, electronic storage devices, or as application
25 services over a network.

[0080] The present invention is believed to be applicable to a variety of systems for performance analysis, and is believed to be beneficial in analyzing the performance of distributed systems. Other aspects and embodiments of the present invention will be apparent to those skilled in the art from consideration of the specification and practice of
30 the invention disclosed herein. It is intended that the specification and illustrated embodiments be considered as examples only, with a true scope and spirit of the invention being indicated by the following claims.